You will remember from our earlier discussion of inviscid flows that steady, uniform, incompressible flow of inviscid fluid past a body resulted in no drag on the body, even though in the real world we know that putting a body in a flow gives us drag. We called this “D’Alembert’s Paradox”. At the time, we learned that the proper handling of drag forces in such a situation requires consideration of viscosity. In class we decomposed the drag force into surface friction, which has to do with viscous shear stresses, and pressure forces. It’s obvious, then, that the viscosity of a fluid leads to surface friction. What’s not so obvious is that the fluid viscosity also explains the pressure forces. The explanation for this lies in the concept of boundary layer separation that we talked about earlier in the context of the first lab.

We’ll start by looking at the mechanisms by which a boundary layer either remains ‘attached’ to a surface, or separates. A boundary layer can remain ‘attached’ to a surface if the flow streamlines in the boundary layer have no difficulty following the surface contours. To understand what might cause the boundary layer to be unable to follow the surface contours, consider Fig. 1, depicting the boundary layer on a flat plate, where the mean flow is from left to right. In the leftmost cartoon, the flow is moving in the positive x-direction, pushed along by the negative pressure gradient, \( dp/dx < 0 \). The velocity gradient at the surface, \( du/dy (y = 0) \), is positive. Now suppose we increase the pressure gradient (i.e. make it more positive). As the pressure gradient changes sign and becomes positive, it will want to push the flow in the negative x-direction. If the flow has some inertia, it will tend to push against this adverse pressure gradient. At some point, these effects will be in balance, and the situation in the middle cartoon in Fig. 1 will pertain, where \( du/dy \) at the surface is zero. As the pressure gradient is increased past this point, we get flow reversal near the surface (the cartoon on the right). When this happens, the left-to-right boundary layer flow can no longer follow the contour of the surface. Instead, the boundary layer is diverted upward to go around the region of reversed flow. This process is called boundary layer separation. The reason why the separation effect is felt initially in the boundary layer is because the fluid is slower, and thus has less momentum, near the surface. The figure accordingly shows the effect of the changing pressure gradient as being most pronounced near the surface.

The favorite example of fluids instructors and textbooks everywhere for illustrating boundary layer separation is flow around a sphere. Figure 2 shows schematically what happens if the fluid is inviscid. As the flow moves around the front half of the sphere, it accelerates (basically, the fluid has to fit through a smaller space), and as according to the Bernoulli equation, the pressure...
decreases. Then, as the flow moves past the back half of the sphere, the velocity decreases and the pressure increases until both return to their initial values. The velocity and pressure profiles are symmetric between the front and back of the sphere, because (in the absence of viscosity) there is no friction-type effect to sap energy from the flow. In particular, the symmetry means that there is no pressure drag on the sphere (part of what D’Alembert noted in his paradox).

Now let’s consider what happens if the viscosity of the fluid is non-zero. (We have to be a little more careful in our reasoning here, since we know that Bernoulli’s equation doesn’t apply in regions where viscosity is significant.) This situation is depicted in Fig. 3. There is now a boundary layer at the sphere’s surface. What we’re interested in is the pressure in the boundary layer, since we saw above that the pressure gradient in the boundary layer controls the flow separation process. To find the boundary layer pressure, we first note that one definition of the boundary layer is that it’s the part of the flow where viscosity is important. This means that any streamlines that are outside the boundary layer can be analyzed assuming that the fluid is inviscid. Then, we recall from our discussion of boundary layers that the pressure doesn’t change in the direction perpendicular to the flow. This, in turn, means that the pressure that we find for a streamline just outside the boundary layer is the same pressure that pertains inside the boundary layer. We will also note here that the boundary layer will be very thin; in particular, its thickness will be much less than the radius of the sphere.

We determine the pressure curve shown in Fig. 3, then, by applying the same reasoning as in the inviscid case to understand how the pressure (and velocity) vary as we move from the front of the sphere to the back, in a streamline just outside the boundary layer, then extrapolating the
resulting pressures to the boundary layer itself. On the front half of the sphere, the pressure at the surface is decreasing as we move in the flow direction, so according to the examples of Fig. 1, the boundary layer flows happily along the surface of the sphere. However, as we move to the back half of the sphere, the pressure at the surface increases as we move in the flow direction. Eventually, the boundary layer will be unable to fight against this adverse pressure gradient, and the flow will separate. After that, the pressure values essentially freeze; that is, the pressures (and velocities) don’t go on to recover to their initial values, because once the streamlines separate, the effective area through which the fluid flows is fixed, so the velocities don’t change as the flow moves downstream.

The result is that net pressure on the front of the sphere is higher than that on the back of the sphere, meaning that there is a pressure drag on the sphere (as everyone is amply familiar with in the real world). A good illustration of this boundary layer separation phenomenon is given in Fig. 4.

As is well known, the pressure drag on a sphere is lowered when the boundary layer flow is turbulent. This is because a turbulent boundary layer is better able to overcome the adverse pressure gradient. (Recall that turbulence introduces an additional mechanism (besides viscosity) by which momentum can be transported in a fluid, and as result, high-momentum fluid can get closer to the wall in a turbulent boundary layer than in a laminar boundary layer.) The boundary layer can be caused to become turbulent if the sphere’s surface is somehow rough – examples are the dimples on a golf ball or the raised seams on a baseball. In Fig. 5, the boundary layer is made turbulent by a wire placed around the sphere upstream of its equator. By comparing Fig. 5 with Fig. 4, it is clear that the turbulent boundary layer gets further around the sphere before

Figure 4: Boundary layer separation on a sphere (from van Dyke, *An Album of Fluid Motion*.)

Figure 5: A turbulent boundary layer on a sphere (from van Dyke).
separating. The result is that the pressure recovers better around the back of the sphere, so the pressure on the back of the sphere is higher for the turbulent boundary layer as compared with the laminar boundary layer.

Experimental results for the drag on a smooth sphere are given in Fig. 6. (This plot is qualitatively similar to the cylinder drag plot given in the text as Fig. 9.12.) At the laminar/turbulent transition, the drag suddenly drops by a factor of five, consistent with the above discussion. This plot is for a smooth sphere; it stands to reason that by making the sphere surface rough, we can ‘trip’ the boundary layer into turbulence, with the effect that the laminar/turbulent transition point on Fig. 6 gets moved to the left. That is, for a given fluid velocity, $V$, a rough-surfaced sphere will have lower drag than a smooth sphere of the same diameter. This is why golf balls have dimples on them.

A couple of points to make:

- The effects of pressure drag, such as the relation of flow separation on sphere drag, and friction drag, such as the drag on a flat plate parallel to a flow, differ in one obvious respect; their dependence on whether the flow is laminar or turbulent. We saw earlier that the friction drag on a flat plate increased when the boundary became turbulent, whereas the pressure drag on a sphere decreases when the boundary layer becomes turbulent.

- The notion that the pressure and velocity ‘recover’ as the flow moves around a sphere is attractive, but as usual with intuitive ideas, we have to be careful that any conclusions drawn match with experimental observations. For example, we might reasonably expect that the same physical reasoning we’ve used here pertains when we talk about lift of bodies (wings, for example) in viscous flows instead of drag. It turns out that we need to be careful when we attempt to reason through the properties of airfoil lift – we’ll get to this later.

Figure 6: Drag coefficient, $C_D \equiv \frac{F_D}{\frac{1}{2}\rho V^2 A}$, vs. Reynolds number, $Re = \frac{\rho V d}{\mu}$, for a sphere. Here $F_D$ is the net drag force, $d$ is the sphere diameter and $A$ is the sphere frontal area, $A = \pi d^2/4$. The drag coefficient drops by a factor of 5 upon the transition to a turbulent boundary layer. (From Munson, Young & Okiishi, *Fundamentals of Fluid Mechanics.*)